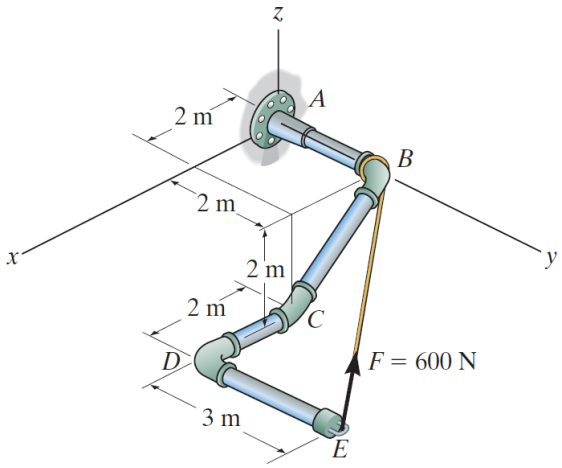
Problem 2-113

Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



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Solution

Write the position vectors to the points B, D, and E.

$$\mathbf{r}_B = \langle 0, 2, 0 \rangle \text{ m}$$

$$\mathbf{r}_D = \langle 4, 2, -2 \rangle \text{ m}$$

$$\mathbf{r}_E = \langle 4, 5, -2 \rangle \text{ m}$$

The position vector from E to B is then

$$\begin{aligned}\mathbf{r}_{EB} &= \mathbf{r}_B - \mathbf{r}_E \\ &= \langle -4, -3, 2 \rangle \text{ m}.\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{EB}| &= \sqrt{(-4)^2 + (-3)^2 + 2^2} \text{ m} \\ &= \sqrt{29} \text{ m}.\end{aligned}$$

Divide $\mathbf{r}_{EB}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{EB} = \frac{\mathbf{r}_{EB}}{|\mathbf{r}_{EB}|} = \frac{\langle -4, -3, 2 \rangle}{\sqrt{29}}$$

The force $\mathbf{F}$ can now be written.

$$\mathbf{F} = F\hat{\mathbf{u}}_{EB} = 600\,\frac{\langle -4, -3, 2 \rangle}{\sqrt{29}} \text{ N} \approx \langle -446, -334, 223 \rangle \text{ N}$$

Write the position vector going from E to D.

$$\begin{aligned}\mathbf{r}_{ED} &= \mathbf{r}_D - \mathbf{r}_E \\ &= \langle 0, -3, 0\rangle \text{ m}\end{aligned}$$

The unit vector going from E to D is then

$$\hat{\mathbf{u}}_{ED} = \frac{\mathbf{r}_{ED}}{|\mathbf{r}_{ED}|} = \frac{\langle 0, -3, 0\rangle}{3} = \langle 0, -1, 0\rangle.$$

Take the dot product of $\mathbf{F}$ with this unit vector to find the component of $\mathbf{F}$ along segment DE.

$$F_{\parallel} = \mathbf{F} \cdot \hat{\mathbf{u}}_{ED} = \left(600\,\frac{\langle -4, -3, 2\rangle}{\sqrt{29}}\text{ N}\right) \cdot \langle 0, -1, 0\rangle = \frac{600}{\sqrt{29}}(-3)(-1)\text{ N} \approx 334\text{ N}$$

Therefore, the magnitude of this parallel component is

$$|F_{\parallel}| \approx 334\text{ N}.$$

Subtract the component of $\mathbf{F}$ parallel to DE from $\mathbf{F}$ to get the component of $\mathbf{F}$ perpendicular to DE.

$$\begin{aligned}\mathbf{F}_{\perp} &= \mathbf{F} - \mathbf{F}_{\parallel} \\ &= F\hat{\mathbf{u}}_{EB} - |F_{\parallel}|\hat{\mathbf{u}}_{ED} \\ &\approx 600\,\frac{\langle -4, -3, 2\rangle}{\sqrt{29}}\text{ N} - \frac{600}{\sqrt{29}}(3)\,\langle 0, -1, 0\rangle \\ &\approx \langle -446, 0, 223\rangle\text{ N}\end{aligned}$$

Therefore, the magnitude of this perpendicular component is

$$\begin{aligned}|\mathbf{F}_{\perp}| &\approx \sqrt{(-446)^2 + (223)}\text{ N} \\ &\approx 498\text{ N}.\end{aligned}$$